

LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – PHYSICS

THIRD SEMESTER – NOVEMBER 2007

MT 3102/3100 - MATHEMATICS FOR PHYSICS

AB 3

Date : 02/11/2007
Time : 9:00 - 12:00

Dept. No.

Max. : 100 Marks

SECTION A

Answer **ALL** the questions.

10 × 2 = 20

1. Sum the series: $(\log a) - \frac{1}{2!}(\log a)^2 + \frac{1}{3!}(\log a)^3 + \dots \infty$
2. Define a Hermitian matrix and give an example.
3. Find the inverse of an orthogonal matrix $\frac{1}{3} \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$.
4. At any point on the curve $y = b \sin \frac{x}{a}$, find the lengths of the subtangent and subnormal.
5. Find the slope of the curve $r = a(1 - \cos \theta)$ at $\theta = \frac{\pi}{2}$.
6. Solve approximately: $\cos\left(\frac{\pi}{3} + \theta\right) = 0.49$
7. Prove that $\sinh^{-1} x = \log_e(x + \sqrt{x^2 + 1})$.
8. Find $L(\cosh 3t)$.
9. Find the inverse Laplace transform of $\frac{1}{(s+2)^2 + 16}$.
10. From the well-shuffled pack of 52 cards, one card is drawn at random. What is the probability that it will be (i) a jack and (ii) a spade?

SECTION B

Answer **ANY FIVE** questions.

5 × 8 = 40

11. Prove: $2 \left\{ \frac{1}{2n+1} + \frac{1}{3} \cdot \frac{1}{(2n+1)^3} + \frac{1}{5} \cdot \frac{1}{(2n+1)^5} + \dots \right\} = \log \left(\frac{n+1}{n} \right)$
12. Determine the characteristic roots of the matrix $\begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & -1 \\ 2 & -1 & 0 \end{pmatrix}$.
13. If $y = \sin(m \sin^{-1} x)$, prove that $(1 - x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$.
14. Find the maxima and minima of the function $2x^3 - 3x^2 - 36x + 10$.

15. Evaluate: $\lim_{x \rightarrow 0} \frac{5 \sin x - \sin 5x}{5(\cos x - \cos 5x)}$

16. Evaluate: (i) $L\left(\frac{\cos 2t - \cos 3t}{t}\right)$ and (ii) $L^{-1}\left(\frac{s}{(s+2)^2}\right)$

17. Find the Laplace transform of $f(t) = \begin{cases} 1 & ; 0 < t < b \\ -1 & ; b < t < 2b \end{cases}$.

18. Find the mean, variance and standard deviation of the following distribution:

x	3	8	12
f(x)	1/3	1/2	1/6

SECTION C

Answer **ANY TWO** questions:

2 × 20 = 40

19. (a) When x is small, show that $\sqrt{x^2 + 4} - \sqrt{x^2 + 1} = 1 - \frac{1}{4}x^2 + \frac{7}{64}x^4$ nearly.

(b) Verify Cayley-Hamilton theorem for $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{pmatrix}$. (10+10)

20. (a) Separate $\tan^{-1}(x + iy)$ into real and imaginary parts.

(b) If $u = \log \tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$ show that (i) $\tanh \frac{u}{2} = \tan \frac{\theta}{2}$ (ii) $\theta = -i \log \tan\left(\frac{\pi}{4} + i \frac{u}{2}\right)$. (10+10)

21. (a) Solve: $y'' + 2y' + 5y = 4e^{-t}$ given that $y(0) = y'(0) = 0$

(b) Find the angle of intersection of the curves $r = \frac{a}{1 + \cos \theta}$ and $r = \frac{b}{1 - \cos \theta}$. (10+10)

22. (a) Find the nth derivative of $\frac{x^2}{(x+1)^2(x+2)}$.

(b) The probability that John hits a target is $p = 1/4$. He fixes $n = 6$ times. Find the probability that he hits the target: (i) exactly 2 times (ii) more than 4 times (iii) at least once. (10+10)
